

## Ejercicios resueltos

1. Calcular los siguientes límites algebraicos

$$1) \lim_{x \rightarrow 2} \frac{x^2 + 1}{x^2 - 1} = \frac{2^2 + 1}{2^2 - 1} = \frac{5}{3}$$

$$2) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - x} = \frac{1^2 - 2 \cdot 1 + 1}{1^3 - 1} = \frac{0}{0} \text{ pero } \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - x} = \lim_{x \rightarrow 1} \frac{(x-1)^2}{x(x^2-1)} = \\ \lim_{x \rightarrow 1} \frac{(x-1)^2}{x(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{(x-1)}{x(x+1)} = \frac{1-1}{1(2)} = \frac{0}{2} = 0$$

$$3) \lim_{x \rightarrow 1} \frac{(x-1)\sqrt{2-x}}{1-x^2} = \frac{0}{0}, \text{ pero } \lim_{x \rightarrow 1} \frac{(x-1)\sqrt{2-x}}{1-x^2} = \lim_{x \rightarrow 1} \frac{(x-1)\sqrt{2-x}}{(1-x)(1+x)} = \\ \lim_{x \rightarrow 1} \frac{-(1-x)\sqrt{2-x}}{(1-x)(1+x)} = \lim_{x \rightarrow 1} \frac{-1\sqrt{2-x}}{1+x} = \frac{-1}{2}$$

$$4) \lim_{x \rightarrow 1} \frac{1}{1-x} - \frac{3}{1-x^3} \lim_{x \rightarrow 1} \frac{1}{1-x} - \frac{3}{1-x^3} = \infty - \infty \text{ que es una forma indeterminada.}$$

$$\text{Pero } \lim_{x \rightarrow 1} \frac{1}{1-x} - \frac{3}{1-x^3} = \lim_{x \rightarrow 1} \frac{1}{1-x} - \frac{3}{(1-x)(1+x+x^2)} = \\ \lim_{x \rightarrow 1} \frac{(1+x+x^2)}{1-x} - \frac{3}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(1+x+x^2-3)}{(1-x)(1+x+x^2)} = \\ \lim_{x \rightarrow 1} \frac{(x^2+x-2)}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{(1-x)(1+x+x^2)} = \\ \lim_{x \rightarrow 1} \frac{-(1-x)(x+2)}{(1-x)(1+x+x^2)} = \lim_{x \rightarrow 1} \frac{-(x+2)}{(1+x+x^2)} = \frac{-3}{3} = -1$$

$$5) \lim_{x \rightarrow 1} \frac{x+2}{x^2-5x+4} + \frac{x-4}{3(x^2-3x+2)} = \lim_{x \rightarrow 1} \frac{x+2}{(x-4)(x-1)} + \frac{x-4}{3(x-2)(x-1)} = \\ \lim_{x \rightarrow 1} \frac{3(x+2)(x-2)+(x-4)(x-4)}{3(x-4)(x-2)(x-1)} = \lim_{x \rightarrow 1} \frac{3(x^2-4)+(x-4)^2}{3(x-4)(x-2)(x-1)} = \\ \lim_{x \rightarrow 1} \frac{3x^2-12+x^2-8x+16}{3(x-4)(x-2)(x-1)} = \lim_{x \rightarrow 1} \frac{4x^2-8x+4}{3(x-4)(x-2)(x-1)} = \\ \lim_{x \rightarrow 1} \frac{4(x^2-2x+1)}{3(x-4)(x-2)(x-1)} = \lim_{x \rightarrow 1} \frac{4(x^2-2x+1)}{3(x-4)(x-2)(x-1)} = \\ \lim_{x \rightarrow 1} \frac{4(x-1)^2}{3(x-4)(x-2)(x-1)} = \lim_{x \rightarrow 1} \frac{4(x-1)}{3(x-4)(x-2)} = \frac{0}{-24} = 0$$

$$6) \lim_{x \rightarrow \infty} \frac{2+x-3x^3}{5-x^2+3x^3} = \lim_{x \rightarrow \infty} \frac{\frac{2+x-3x^3}{x^3}}{\frac{5-x^2+3x^3}{x^3}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x^3} + \frac{x}{x^3} - \frac{3x^3}{x^3}}{\frac{5}{x^3} - \frac{x^2}{x^3} + \frac{3x^3}{x^3}} = \frac{0+0-3}{0-0+3} = -1$$

$$7) \lim_{x \rightarrow \infty} \frac{x^3}{x^2+1} - x = \lim_{x \rightarrow \infty} \frac{x^3 - x(x^2+1)}{x^2+1} = \lim_{x \rightarrow \infty} \frac{x^3 - x^3 - x}{x^2+1} = \lim_{x \rightarrow \infty} \frac{-x}{x^2+1} = \\ \lim_{x \rightarrow \infty} \frac{-\frac{x}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{0}{1+0} = 0$$

$$8) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - 3\sqrt{x}}{\sqrt[4]{x^3+5x} - x} = \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{x^2+1}-3\sqrt{x}}{x}}{\frac{\sqrt[4]{x^3+5x}-x}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{x^2+1}}{x} - \frac{3\sqrt{x}}{x}}{\frac{\sqrt[4]{x^3+5x}}{x} - \frac{x}{x}} \\ = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2+1}{x^2}} - \frac{3}{x^{1/2}}}{\sqrt[4]{\frac{x^3+5x}{x^4}} - 1} = \lim_{x \rightarrow \infty} \frac{\sqrt{1+\frac{1}{x^2}} - \frac{3}{\sqrt{x}}}{\sqrt[4]{\frac{1}{x} + \frac{5}{x^3}} - 1} = \frac{\sqrt{1+0}-0}{\sqrt[4]{0+0}-1} = -1$$

$$9) \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - 1}{x} \text{ Racionalizando obtenemos}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - 1}{x} \frac{\sqrt{1+x^2} + 1}{\sqrt{1+x^2} + 1} = \lim_{x \rightarrow 0} \frac{1+x^2 - 1}{x(\sqrt{1+x^2} + 1)} = \\ \lim_{x \rightarrow 0} \frac{x^2}{x(\sqrt{1+x^2} + 1)} = \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x^2} + 1} = \frac{0}{2} = 0$$

$$10) \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5} \text{ Racionalizando obtenemos}$$

$$\lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5} \frac{\sqrt{x-1} + 2}{\sqrt{x-1} + 2} = \lim_{x \rightarrow 5} \frac{x-1-4}{(x-5)(\sqrt{x-1} + 2)} = \lim_{x \rightarrow 5} \frac{x-5}{(x-5)(\sqrt{x-1} + 2)} = \\ \lim_{x \rightarrow 5} \frac{1}{\sqrt{x-1} + 2} = \frac{1}{4}$$

$$11) \lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2}}{x-1} = \lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2} + 2 - 2}{x-1} =$$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3} - 2}{x-1} - \frac{\sqrt{3+x^2} - 2}{x-1} \text{ Racionalizando cada fracción obtenemos:}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3} - 2}{x-1} \frac{\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4}{\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4} - \frac{\sqrt{3+x^2} - 2}{x-1} \frac{\sqrt{3+x^2} + 2}{\sqrt{3+x^2} + 2} =$$

$$\lim_{x \rightarrow 1} \frac{(7+x^3)-8}{(x-1)\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4} - \frac{3+x^2-4}{(x-1)(\sqrt{3+x^2} + 2)} =$$

$$\lim_{x \rightarrow 1} \frac{x^3-1}{(x-1)\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4} - \frac{x^2-1}{(x-1)(\sqrt{3+x^2} + 2)} =$$

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4} - \frac{(x-1)(x+1)}{(x-1)(\sqrt{3+x^2} + 2)} = \\ & \lim_{x \rightarrow 1} \frac{(x^2+x+1)}{\sqrt[3]{(7+x^3)^2} + 2\sqrt[3]{7+x^3} + 4} - \frac{(x+1)}{(\sqrt{3+x^2} + 2)} = \\ & \frac{1+1+1}{4+4+4} - \frac{2}{2+2} = \frac{3}{12} - \frac{2}{4} = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4} \end{aligned}$$

12)  $\lim_{x \rightarrow \infty} \sqrt{(x+m)(x+n)} - x$  Racionalizando queda:

$$\lim_{x \rightarrow \infty} \sqrt{(x+m)(x+n)} - x \cdot \frac{\sqrt{(x+m)(x+n)} + x}{\sqrt{(x+m)(x+n)} + x} = \lim_{x \rightarrow \infty} \frac{(x+m)(x+n) - x^2}{\sqrt{(x+m)(x+n)} + x} =$$

$$\lim_{x \rightarrow \infty} \frac{x^2 + (m+n)x + mn - x^2}{\sqrt{(x+m)(x+n)} + x} = \lim_{x \rightarrow \infty} \frac{(m+n)x + mn}{\sqrt{x^2 + (m+n)x + mn} + x} \text{ Dividiendo por la}$$

potencia más grande de  $x$ , que es  $x^2$  queda:

$$\lim_{x \rightarrow \infty} \frac{(m+n) + \frac{mn}{x}}{\sqrt{1 + \frac{(m+n)}{x} + \frac{mn}{x^2}} + 1} = \frac{(m+n) + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{m+n}{2}$$

13)  $\lim_{x \rightarrow 2} \frac{1}{2-x} - \frac{3}{8-x^3} = \lim_{x \rightarrow 2} \frac{1}{2-x} - \frac{3}{(2-x)(4+2x+x^2)} = \lim_{x \rightarrow 2} \frac{4+2x+x^2-3}{(2-x)(4+2x+x^2)} =$

$$\lim_{x \rightarrow 2} \frac{x^2+2x+1}{(2-x)(4+2x+x^2)} = \frac{4+4+1}{0} = \infty$$

14)  $\lim_{x \rightarrow \frac{1}{2}} \frac{8x^2-1}{6x^2-5x+1} = \frac{1}{0} = \infty$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{8x^2-2x-1}{6x^2-5x+1} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x-1)(4x+1)}{(3x-1)(2x-1)} = \lim_{x \rightarrow \frac{1}{2}} \frac{4x+1}{3x-1} = \frac{2+1}{\frac{3}{2}-1} = \frac{3}{\frac{1}{2}} = 6$$

15)  $\lim_{x \rightarrow \infty} \frac{3x^4-2x+1}{3x^2+6x-2}$  Dividiendo por la potencia más grande de  $x$  que es  $x^4$  queda:

$$\lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x^3} + \frac{1}{x^4}}{\frac{3}{x^2} + \frac{6}{x^3} - \frac{2}{x^4}} = \frac{3}{0} = \infty$$

16)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{\sqrt{x^2+9}-3} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2} \cdot \frac{\sqrt{x^2+9}+3}{\sqrt{x^2+9}+3} =$

$$\lim_{x \rightarrow 0} \frac{(x^2+4-4)(\sqrt{x^2+9}+3)}{(x^2+9-9)(\sqrt{x^2+4}+2)} = \lim_{x \rightarrow 0} \frac{(x^2)(\sqrt{x^2+9}+3)}{(x^2)(\sqrt{x^2+4}+2)} =$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} + 3}{\sqrt{x^2 + 4} + 2} = \frac{\sqrt{0 + 9} + 3}{\sqrt{0 + 4} + 2} = \frac{3 + 3}{2 + 2} = \frac{6}{4} = \frac{3}{2}$$

$$17) \lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{x - 4} = \lim_{x \rightarrow 4} \frac{\sqrt{1+2x} - 3}{x - 4} \cdot \frac{\sqrt{1+2x} + 3}{\sqrt{1+2x} + 3} = \lim_{x \rightarrow 4} \frac{1+2x - 9}{(x-4)(\sqrt{1+2x} + 3)} =$$

$$\lim_{x \rightarrow 4} \frac{2x - 8}{(x-4)(\sqrt{1+2x} + 3)} = \lim_{x \rightarrow 4} \frac{2(x-4)}{(x-4)(\sqrt{1+2x} + 3)} = \lim_{x \rightarrow 4} \frac{2}{\sqrt{1+2x} + 3} =$$

$$\frac{2}{6} = \frac{1}{3}$$

$$18) \lim_{x \rightarrow \infty} \frac{3^x + x}{3^x - 2x} \text{ Primero calcularemos } \lim_{x \rightarrow \infty} \frac{x}{3^x} \text{ usando el teorema del Sandwich.}$$

Sabemos que  $3^x \geq x^3$  Para  $x > 3$  Por lo tanto  $\frac{x}{3^x} \leq \frac{x}{x^3}$  además  $\frac{x}{3^x} \geq \frac{1}{3^x}$  por lo que

podemos afirmar que:  $\frac{1}{3^x} \leq \frac{x}{3^x} \leq \frac{x}{x^3} \Rightarrow \frac{1}{3^x} \leq \frac{x}{3^x} \leq \frac{1}{x^2}$  Aplicando límite

$$\lim_{x \rightarrow \infty} \frac{1}{3^x} \leq \lim_{x \rightarrow \infty} \frac{x}{3^x} \leq \lim_{x \rightarrow \infty} \frac{1}{x^2} \Rightarrow 0 \leq \lim_{x \rightarrow \infty} \frac{x}{3^x} \leq 0 \text{ Por lo que } \lim_{x \rightarrow \infty} \frac{x}{3^x} = 0$$

Dividiendo el límite que queremos calcular por  $3^x$  queda:

$$\lim_{x \rightarrow \infty} \frac{3^x + x}{3^x - 2x} = \lim_{x \rightarrow \infty} \frac{1 + \frac{x}{3^x}}{1 - \frac{2x}{3^x}} = \lim_{x \rightarrow \infty} \frac{1 + \frac{x}{3^x}}{1 - 2\frac{x}{3^x}} = \lim_{x \rightarrow \infty} \frac{1 + 0}{1 - 2(0)} = 1$$

$$19) \lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^2 + 1}}{x - 2} \text{ Dividiendo por la potencia más grande de } x \text{ que es } x \text{ queda:}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt[3]{\frac{x^2}{x^3} + \frac{1}{x^3}}}{\frac{x}{x} - \frac{2}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt[3]{\frac{1}{x} + \frac{1}{x^3}}}{1 - \frac{2}{x}} = \frac{0}{1} = 0$$

$$20) \lim_{x \rightarrow \infty} \frac{2x^2 + 1}{\sqrt[3]{x^7 + 2x}} \text{ Dividiendo por la potencia más grande de } x \text{ que es } x^{7/3} \text{ queda:}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{2x^2}{x^{7/3}} + \frac{1}{x^{7/3}}}{\sqrt[3]{\frac{x^7}{x^7} + \frac{2x}{x^7}}} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x^{1/3}} + \frac{1}{x^{7/3}}}{\sqrt[3]{1 + \frac{2}{x^6}}} = \frac{0}{1} = 0$$

$$21) \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + a^2} - a}{\sqrt{x^2 + b^2} - b} = \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + a^2} - a}{\sqrt{x^2 + b^2} - b} \cdot \frac{\sqrt{x^2 + a^2} + a}{\sqrt{x^2 + a^2} + a} \cdot \frac{\sqrt{x^2 + b^2} + b}{\sqrt{x^2 + b^2} + b} =$$

$$\lim_{x \rightarrow 0} \frac{(x^2 + a^2 - a^2)(\sqrt{x^2 + b^2} + b)}{(x^2 + b^2 - b^2)(\sqrt{x^2 + a^2} + a)} = \lim_{x \rightarrow 0} \frac{x^2(\sqrt{x^2 + b^2} + b)}{x^2(\sqrt{x^2 + a^2} + a)} =$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + b^2} + b}{\sqrt{x^2 + a^2} + a} = \frac{\sqrt{b^2} + b}{\sqrt{a^2} + a} = \frac{2b}{2a} = \frac{b}{a}$$

22)  $\lim_{x \rightarrow 1} \frac{1 - \sqrt[3]{x}}{1 - \sqrt{x}}$  Sea  $u^6 = x$  Si  $x \rightarrow 1$  entonces  $u^6 = 1 \Rightarrow u \rightarrow 1$

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{1 - \sqrt[3]{x}}{1 - \sqrt{x}} &= \lim_{u \rightarrow 1} \frac{1 - \sqrt[3]{u^6}}{1 - \sqrt{u^6}} = \lim_{u \rightarrow 1} \frac{1 - u^2}{1 - u^3} = \lim_{u \rightarrow 1} \frac{(1-u)(1+u)}{(1-u)(1+u+u^2)} = \\ \lim_{u \rightarrow 1} \frac{1+u}{1+u+u^2} &= \frac{2}{3}\end{aligned}$$

2. Calcular los siguientes límites trigonométricos

1)  $\lim_{x \rightarrow 0} \frac{\sin 8x}{x} = \lim_{x \rightarrow 0} \frac{\sin 8x}{x} \frac{8}{8} = \lim_{x \rightarrow 0} 8 \frac{\sin 8x}{8x} = (8)(1) = 8$

2)  $\lim_{x \rightarrow 0} \frac{2\sin(3x)}{5x} = \lim_{x \rightarrow 0} \frac{2\sin(3x)}{5x} \frac{3}{3} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \frac{6}{5} = (1) \left(\frac{6}{5}\right) = \frac{6}{5}$

3)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(2x)} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{\sin(2x)} \frac{2x}{3x} \frac{3}{2} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \frac{2x}{\sin(2x)} \frac{3}{2} = (1)(1) \left(\frac{3}{2}\right) = \frac{3}{2}$

4)  $\lim_{x \rightarrow 0} \frac{\tan(2x)}{\tan(5x)} = \lim_{x \rightarrow 0} \frac{\tan(2x)}{\tan(5x)} \frac{5x}{2x} \frac{2}{5} = \lim_{x \rightarrow 0} \frac{\tan(2x)}{2x} \frac{5x}{\tan(5x)} \frac{2}{5} = (1)(1) \left(\frac{2}{5}\right) = \frac{2}{5}$

5)  $\lim_{x \rightarrow 0} \frac{2x}{\sin(9x)} = \lim_{x \rightarrow 0} \frac{2x}{\sin(9x)} \frac{7}{7} = \lim_{x \rightarrow 0} \frac{9x}{\sin(9x)} \frac{2}{9} = (1) \left(\frac{2}{9}\right) = \frac{2}{9}$

6)  $\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} + \frac{1}{2} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{2x} \frac{3}{3} + \frac{1}{2} = \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \frac{3}{2} + \frac{1}{2} = (1) \left(\frac{3}{2}\right) + \frac{1}{2} = \frac{4}{2} = 2$

7)  $\lim_{x \rightarrow 0} \frac{\cos(2x) - \cos(5x)}{x^2} = \lim_{x \rightarrow 0} \frac{-2\sin\left(\frac{2x+5x}{2}\right) \sin\left(\frac{2x-5x}{2}\right)}{x^2} =$   
 $\lim_{x \rightarrow 0} \frac{-2\sin\left(\frac{7x}{2}\right) \sin\left(\frac{-3x}{2}\right)}{x^2} = \lim_{x \rightarrow 0} \frac{\sin\left(\frac{7x}{2}\right) \sin\left(\frac{-3x}{2}\right)}{\frac{-x^2}{2}} =$

$$\lim_{x \rightarrow 0} \frac{\sin\left(\frac{7x}{2}\right)}{x} \frac{\sin\left(\frac{-3x}{2}\right)}{\frac{-x}{2}} = \lim_{x \rightarrow 0} \frac{7}{2} \frac{\sin\left(\frac{7x}{2}\right)}{\frac{7x}{2}} \frac{3\sin\left(\frac{-3x}{2}\right)}{\frac{-3x}{2}} = \frac{7}{2} (1)(3)(1) = \frac{21}{2}$$

8)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\tan x}}{\sin(2x)} = \lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\tan x}}{\sin(2x)} \frac{\sqrt{1+\sin x} + \sqrt{1-\tan x}}{\sqrt{1+\sin x} + \sqrt{1-\tan x}} =$

$$\lim_{x \rightarrow 0} \frac{1 + \sin x - (1 - \tan x)}{(\sin(2x))(\sqrt{1+\sin x} + \sqrt{1-\tan x})} = \lim_{x \rightarrow 0} \frac{\sin x + \tan x}{(\sin(2x))(\sqrt{1+\sin x} + \sqrt{1-\tan x})} =$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\frac{\sin x + \frac{\sin x}{\cos x}}{\cos x}}{(\sin(2x))(\sqrt{1+\sin x} + \sqrt{1-\tan x})} &= \lim_{x \rightarrow 0} \frac{\frac{\sin x \cos x + \sin^2 x}{\cos x}}{(\sin(2x))(\sqrt{1+\sin x} + \sqrt{1-\tan x})} = \\ \lim_{x \rightarrow 0} \frac{\frac{\sin x (\cos x + 1)}{\cos x}}{(2\sin x \cos x)(\sqrt{1+\sin x} + \sqrt{1-\tan x})} &= \\ \lim_{x \rightarrow 0} \frac{\frac{\sin x (\cos x + 1)}{2\sin x \cos^2 x}}{(\sqrt{1+\sin x} + \sqrt{1-\tan x})} &= \lim_{x \rightarrow 0} \frac{\cos x + 1}{(2\cos^2 x)(\sqrt{1+\sin x} + \sqrt{1-\tan x})} = \\ \frac{\cos 0 + 1}{(2\cos^2 0)(\sqrt{1+\sin 0} + \sqrt{1-\tan 0})} &= \frac{1+1}{2(1)(\sqrt{1+0} + \sqrt{1-0})} = \frac{2}{4} = \frac{1}{2} \end{aligned}$$

9)  $\lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{1}{\tan x} = \lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{1}{\frac{\sin x}{\cos x}} = \lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} =$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} \frac{1 + \cos x}{1 + \cos x} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x(1 + \cos x)} =$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x} = \frac{0}{2} = 0$$

10)  $\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\pi}{2} - x \right) \tan x$   
 Sea  $u = \frac{\pi}{2} - x \Rightarrow x = \frac{\pi}{2} - u$ . Si  $x \rightarrow \frac{\pi}{2}$  entonces  $u \rightarrow 0$

$$\lim_{x \rightarrow \frac{\pi}{2}} \left( \frac{\pi}{2} - x \right) \tan x = \lim_{u \rightarrow 0} u \tan \left( \frac{\pi}{2} - u \right) = \lim_{u \rightarrow 0} u \frac{\sin \left( \frac{\pi}{2} - u \right)}{\cos \left( \frac{\pi}{2} - u \right)} =$$

$$\lim_{u \rightarrow 0} u \frac{\sin \frac{\pi}{2} \cos u - \sin u \cos \frac{\pi}{2}}{\cos \frac{\pi}{2} \cos u + \sin \frac{\pi}{2} \sin u} = \lim_{u \rightarrow 0} u \frac{\cos u}{\sin u} = \lim_{u \rightarrow 0} \cos u \frac{u}{\sin u} = (1)(1) = 1$$

11)  $\lim_{x \rightarrow 0} \frac{3x - \arcsen x}{3x + \arctg x}$  Por infinitesimales sabemos que  $\arcsen x \approx x$  y  $\arctg x \approx x$  por lo que:

$$\lim_{x \rightarrow 0} \frac{3x - \arcsen x}{3x + \arctg x} = \lim_{x \rightarrow 0} \frac{3x - x}{3x + x} = \lim_{x \rightarrow 0} \frac{2x}{4x} = \frac{2}{4} = \frac{1}{2}$$

12)  $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}} =$

$$\lim_{x \rightarrow 0} \frac{2 - (1 + \cos x)}{(\sin^2 x)(\sqrt{2} + \sqrt{1 + \cos x})} = \lim_{x \rightarrow 0} \frac{1 - \cos x}{(\sin^2 x)(\sqrt{2} + \sqrt{1 + \cos x})} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{(\sin^2 x)(\sqrt{2} + \sqrt{1 + \cos x})} \frac{1 + \cos x}{1 + \cos x} = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{(1 + \cos x)(\sin^2 x)(\sqrt{2} + \sqrt{1 + \cos x})} =$$

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{(1 + \cos x)(\sin^2 x)(\sqrt{2} + \sqrt{1 + \cos x})} = \lim_{x \rightarrow 0} \frac{1}{(1 + \cos x)(\sqrt{2} + \sqrt{1 + \cos x})} =$$

$$\frac{1}{(2)(\sqrt{2} + \sqrt{2})} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$$

$$13) \lim_{x \rightarrow 0} \frac{\cos(a+x) - \cos(a-x)}{x} = \lim_{x \rightarrow 0} \frac{-2\sin\left(\frac{(a+x)+(a-x)}{2}\right) \sin\left(\frac{(a+x)-(a-x)}{2}\right)}{x} =$$

$$\lim_{x \rightarrow 0} \frac{-2\sin\left(\frac{2a}{2}\right) \sin\left(\frac{2x}{2}\right)}{x} = \lim_{x \rightarrow 0} \frac{-2\sin(a) \sin(x)}{x} = \lim_{x \rightarrow 0} -2\sin(a) \frac{\sin(x)}{x} =$$

$$14) \lim_{x \rightarrow 0} \cosec x - \cot g x = \lim_{x \rightarrow 0} \frac{1}{\sin x} - \frac{1}{\tan x} = 0 \text{ Resuelto con anterioridad.}$$

$$15) \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot g x - \tan x}{\sin x - \cos x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\sin x - \cos x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\sin x - \cos x} =$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{(\sin x - \cos x)(\sin x \cos x)} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x - \cos x)(\sin x \cos x)} =$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{-(\cos x - \sin x)}{\sin x \cos x} = \frac{-(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2})}{\frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2}} = 0$$

$$16) \lim_{x \rightarrow 0} \frac{\cos(2x) + \cot g x + \sin x}{\cos x} \text{ Indeterminado. ¿Mal copiado?}$$

$$17) \lim_{x \rightarrow 0} \frac{\cos(2x) - \sin(\frac{\pi}{2} - x)}{x} = \lim_{x \rightarrow 0} \frac{\cos(2x) - \cos x}{x} = \lim_{x \rightarrow 0} \frac{\cos^2 x - \sin^2 x - \cos x}{x} =$$

$$\lim_{x \rightarrow 0} \frac{\cos x(\cos x - 1) - \sin^2 x}{x} = \lim_{x \rightarrow 0} \frac{\cos x(\cos x - 1) - \sin^2 x}{x} \frac{\cos x + 1}{\cos x + 1} =$$

$$\lim_{x \rightarrow 0} \frac{\cos x(\cos^2 x - 1) - (\cos x + 1)\sin^2 x}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \frac{\cos x(\sin^2 x) - (\cos x + 1)\sin^2 x}{x(\cos x + 1)} =$$

$$\lim_{x \rightarrow 0} \frac{(\sin^2 x)(\cos x - (\cos x + 1))}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-(\sin^2 x)}{x(\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-\sin x}{x} \frac{\sin x}{\cos x + 1} =$$

$$(-1) \left( \frac{0}{2} \right) = 0$$

$$18) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{\cos x} - \cot g x}{\sin(2x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{\cos x} - \frac{\cos x}{\sin x}}{\sin(2x)} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{\cos x} - \frac{\cos x}{\sin x}}{2 \sin x \cos x} =$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{\operatorname{sen} x \sqrt{\cos x} - \cos x}{\operatorname{sen} x}}{2 \operatorname{sen} x \cos x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{sen} x \sqrt{\cos x} - \cos x}{2 \operatorname{sen}^2 x \cos x} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{sen} x \sqrt{\cos x}}{2 \operatorname{sen}^2 x \cos x} - \frac{\cos x}{2 \operatorname{sen}^2 x \cos x} =$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{2 \operatorname{sen} x \sqrt{\cos x}} - \frac{1}{2 \operatorname{sen}^2 x} = \frac{1}{0} - \frac{1}{2} = \infty$$

$$19) \lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{tg} x} - \sqrt{1 - \operatorname{tg} x}}{\operatorname{tg}(2x)} = \lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{tg} x} - \sqrt{1 - \operatorname{tg} x}}{\operatorname{tg}(2x)} \frac{\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}}{\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}} =$$

$$\lim_{x \rightarrow 0} \frac{1 + \operatorname{tg} x - (1 - \operatorname{tg} x)}{(\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}) \operatorname{tg}(2x)} = \lim_{x \rightarrow 0} \frac{2 \operatorname{tg} x}{(\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}) \operatorname{tg}(2x)} =$$

$$\lim_{x \rightarrow 0} \frac{2}{\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}} \frac{\operatorname{tg} x}{\operatorname{tg}(2x)} \frac{x}{x} =$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{1 + \operatorname{tg} x} + \sqrt{1 - \operatorname{tg} x}} \frac{\operatorname{tg} x}{x} \frac{2x}{\operatorname{tg}(2x)} = \frac{1}{\sqrt{1 + 0} + \sqrt{1 - 0}} (1)(1) = \frac{1}{2}$$

$$20) \lim_{x \rightarrow 0} \frac{\operatorname{sen}^2 x - 3 \operatorname{tg}^4 x}{\operatorname{sen}^3 x - \operatorname{sen}^2 x} = \lim_{x \rightarrow 0} \frac{(\operatorname{sen}^2 x)(1 - 3(\operatorname{tg}^2 x \sec^2 x))}{\operatorname{sen}^2 x (\operatorname{sen} x - 1)} =$$

$$\lim_{x \rightarrow 0} \frac{1 - 3(\operatorname{tg}^2 x \sec^2 x)}{\operatorname{sen} x - 1} = \frac{1 - 3(0 * 1)}{0 - 1} = -1$$

3. Calcular los siguientes límites exponenciales y logarítmicos

$$1) \lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} [(1 + 2x)^{\frac{1}{2x}}]^2 = e^2$$

$$2) \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = 3 \frac{e^{3x} - 1}{3x} = 3(1) = 3$$

$$3) \lim_{x \rightarrow 0} (1 - 2x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} (1 + (-2x))^{\frac{1}{x}} = \lim_{x \rightarrow 0} [(1 + (-2x))^{\frac{1}{-2x}}]^{-2} = e^{-2}$$

$$4) \lim_{x \rightarrow 0} (1 + 3x)^{\frac{2}{x}} = \lim_{x \rightarrow 0} [(1 + 3x)^{\frac{1}{x}}]^2 = \lim_{x \rightarrow 0} [(1 + 3x)^{\frac{1}{3x}}]^6 = e^6$$

$$5) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{4x} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{1}{x}\right)^x\right]^4 = e^4$$

$$6) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{5x+3} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{1}{2x}\right)^{2x}\right]^{\frac{5x+3}{2x}} =$$

$$\left[ \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{2x} \right]^{\lim_{x \rightarrow \infty} \frac{5x+3}{2x}} = e^{\frac{5}{2}}$$

7)  $\lim_{x \rightarrow 0} \frac{e^{\operatorname{tg} x} - 1}{\operatorname{tg} x}$  Sea  $u = \operatorname{tg} x$  si  $x \rightarrow 0$  entonces  $u \rightarrow 0$  y  $\lim_{x \rightarrow 0} \frac{e^{\operatorname{tg} x} - 1}{\operatorname{tg} x} = \lim_{u \rightarrow 0} \frac{e^u - 1}{u} = 1$

8)  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\operatorname{sen}(3x)} = \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\operatorname{sen}(3x)} \cdot \frac{3x}{2x} \cdot \frac{2}{3} = \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} \cdot \frac{3x}{\operatorname{sen}(3x)} \cdot \frac{2}{3} = (1)(1) \left( \frac{2}{3} \right) = \frac{2}{3}$

9)  $\lim_{x \rightarrow 0} \frac{e^{3x} - e^x}{\operatorname{arctg} x}$  Por infinitesimales sabemos que  $\operatorname{arctg} x \approx x$  entonces

$$\lim_{x \rightarrow 0} \frac{e^{3x} - e^x}{\operatorname{arctg} x} = \lim_{x \rightarrow 0} \frac{e^{3x} - e^x}{x} = \lim_{x \rightarrow 0} \frac{e^{3x} - 1 - (e^x - 1)}{x} =$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} - \frac{e^x - 1}{x} = 3 \frac{e^{3x} - 1}{3x} - \frac{e^x - 1}{x} = 3(1) - 1 = 2$$

10)  $\lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{\operatorname{arctg}(3x) - \operatorname{arctg}(2x)}$  Por infinitesimales  $\operatorname{arctg}(3x) \approx 3x$  y  $\operatorname{arctg}(2x) \approx 2x$

$$\lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{3x - 2x} = \lim_{x \rightarrow 0} \frac{e^{5x} - 1 - (e^{2x} + 1)}{x} = \lim_{x \rightarrow 0} \frac{e^{5x} - 1}{x} - \frac{e^{2x} - 1}{x} =$$

$$\lim_{x \rightarrow 0} 5 \frac{e^{5x} - 1}{5x} - 2 \frac{e^{2x} - 1}{2x} = 5(1) - 2(1) = 3$$

11)  $\lim_{x \rightarrow 0} \frac{\operatorname{arcsen} x - \operatorname{arctg} x}{x^3}$

Sea  $\alpha = \operatorname{arcsen} x$ ,  $\beta = \operatorname{arctg} x$  y  $t = \operatorname{arcsen} x - \operatorname{arctg} x \Rightarrow t = \alpha - \beta \Rightarrow$

$$\operatorname{sen} t = \operatorname{sen}(\alpha - \beta) = \operatorname{sen} \alpha \cos \beta - \cos \alpha \operatorname{sen} \beta$$

Como  $\alpha = \operatorname{arcsen} x \Rightarrow \operatorname{sen} \alpha = x$ ,  $\cos \alpha = \sqrt{1 - x^2}$  y

$$\beta = \operatorname{arctg} x \Rightarrow \operatorname{tg} \beta = x \Rightarrow \operatorname{sen} \beta = \frac{x}{\sqrt{x^2 + 1}} \wedge \cos \beta = \frac{1}{\sqrt{x^2 + 1}} \Rightarrow$$

$$\operatorname{sen} t = x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}} \Rightarrow t = \operatorname{arcsen} \left( x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}} \right)$$

Reemplazando:

$$\lim_{x \rightarrow 0} \frac{\operatorname{arcsen} \left( x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}} \right)}{x^3} \frac{x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}}}{x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}}} =$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{arcsen} \left( x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}} \right)}{x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}}} \frac{x \frac{1}{\sqrt{x^2 + 1}} - \sqrt{1 - x^2} \frac{x}{\sqrt{x^2 + 1}}}{x^3} =$$

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{\arcsen \left( x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}} \right)}{x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}}} \cdot \frac{\frac{x}{\sqrt{x^2+1}}(1-\sqrt{1-x^2})}{x^3} = \\
& \lim_{x \rightarrow 0} \frac{\arcsen \left( x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}} \right)}{x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}}} \cdot \frac{\frac{x}{\sqrt{x^2+1}}(1-\sqrt{1-x^2})}{x^3} \cdot \frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}} = \\
& \lim_{x \rightarrow 0} \frac{\arcsen \left( x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}} \right)}{x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}}} \cdot \frac{\frac{x}{\sqrt{x^2+1}}(x^2)}{x^3(1+\sqrt{1-x^2})} = \\
& \lim_{x \rightarrow 0} \frac{\arcsen \left( x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}} \right)}{x \frac{1}{\sqrt{x^2+1}} - \sqrt{1-x^2} \frac{x}{\sqrt{x^2+1}}} \cdot \frac{\frac{1}{\sqrt{x^2+1}}}{(1+\sqrt{1-x^2})} = (1) \frac{1}{2} = \frac{1}{2}
\end{aligned}$$

$$12) \lim_{x \rightarrow +\infty} \frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \lim_{x \rightarrow +\infty} \frac{2^x - \frac{1}{2^x}}{2^x + \frac{1}{2^x}} = \lim_{x \rightarrow +\infty} \frac{\frac{2^{2x}-1}{2^x}}{\frac{2^{2x}+1}{2^x}} = \lim_{x \rightarrow +\infty} \frac{2^{2x}-1}{2^{2x}+1} = 1$$

$$13) \lim_{x \rightarrow -\infty} \frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \lim_{x \rightarrow +\infty} \frac{2^{-x} - 2^x}{2^{-x} + 2^x} = \lim_{x \rightarrow +\infty} -\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = -1 \text{ Resuelto con anterioridad.}$$

$$\begin{aligned}
14) \lim_{x \rightarrow 0} (\cos x + \operatorname{sen} x)^{\frac{1}{x}} &= \lim_{x \rightarrow 0} (1 - 1 + \cos x + \operatorname{sen} x)^{\frac{1}{x}} = \\
&\lim_{x \rightarrow 0} \left[ (1 + \cos x + \operatorname{sen} x - 1)^{\frac{1}{\cos x + \operatorname{sen} x - 1}} \right]^{\frac{\cos x + \operatorname{sen} x - 1}{x}} = \lim_{x \rightarrow 0} \frac{\cos x - 1 + \operatorname{sen} x}{x} = \\
&\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} + \frac{\operatorname{sen} x}{x} = e^{x \rightarrow 0} \frac{\cos x - 1}{x} \frac{\cos x + 1}{\cos x + 1} + \frac{\operatorname{sen} x}{x} = e^{x \rightarrow 0} \frac{\cos^2 x - 1}{x(\cos x + 1)} + \frac{\operatorname{sen} x}{x} = \\
&e^{x \rightarrow 0} \frac{-\operatorname{sen}^2 x}{x(\cos x + 1)} + \frac{\operatorname{sen} x}{x} = e^{x \rightarrow 0} \frac{\operatorname{sen} x}{x} \frac{-\operatorname{sen} x}{\cos x + 1} + \frac{\operatorname{sen} x}{x} = e^{(1)(0)+1} = e
\end{aligned}$$

$$15) \lim_{x \rightarrow \sqrt{2}} \frac{e^{2x^2+x-1} - e^{x^2+x+1}}{x^2 - 2} = \lim_{x \rightarrow \sqrt{2}} \frac{e^x (e^{2x^2-1} - e^{x^2+1})}{x^2 - 2} \text{ Sea } u = x^2 - 2 \Rightarrow x^2 = u + 2$$

Si  $x \rightarrow \sqrt{2} \Rightarrow u \rightarrow 0$  Reemplazando queda:

$$\lim_{u \rightarrow 0} \frac{e^{\sqrt{u+2}} (e^{2u+3} - e^{u+3})}{u} = \lim_{u \rightarrow 0} \frac{e^{\sqrt{u+2}} e^3 (e^{2u} - e^u)}{u} = \lim_{u \rightarrow 0} \frac{e^{\sqrt{u+2}} e^3 (e^{2u} - 1 - (e^u - 1))}{u} =$$

$$\lim_{u \rightarrow 0} e^{\sqrt{u+2}} e^3 \left( \frac{e^{2u} - 1}{u} - \frac{e^u - 1}{u} \right) = \lim_{u \rightarrow 0} e^{\sqrt{u+2}+3} \left( 2 \frac{e^{2u} - 1}{2u} - \frac{e^u - 1}{u} \right) =$$

$$e^{\sqrt{2}+3} (2(1) - (1)) = e^{\sqrt{2}+3}$$

$$16) \lim_{x \rightarrow 0} \frac{\ln(1-x) + \sin(3x)}{x} = \lim_{x \rightarrow 0} \frac{\ln(1-x)}{x} + \frac{\sin(3x)}{x} = \lim_{x \rightarrow 0} \frac{\ln(1-x)}{x} + \lim_{x \rightarrow 0} 3 \frac{\sin(3x)}{3x}.$$

$$3 + \lim_{x \rightarrow 0} \frac{\ln(1-x)}{x} \text{ Sea } u = \ln(1-x) \Rightarrow e^u = 1-x \Rightarrow x = 1-e^u$$

$$\lim_{x \rightarrow 0} \frac{\ln(1-x)}{x} = \lim_{u \rightarrow 0} \frac{u}{1-e^u} = \lim_{u \rightarrow 0} -\frac{u}{e^u - 1} = \lim_{u \rightarrow 0} -\left(\frac{e^u - 1}{u}\right)^{-1} = -1 \Rightarrow$$

$$\lim_{x \rightarrow 0} \frac{\ln(1-x) + \sin(3x)}{x} = 3 + (-1) = 2$$

$$17) \lim_{x \rightarrow 0} \frac{b^x - a^x}{x} = \lim_{x \rightarrow 0} \frac{b^x - 1 - (a^x - 1)}{x} = \lim_{x \rightarrow 0} \frac{b^x - 1}{x} - \frac{a^x - 1}{x} = \ln b - \ln a = \ln\left(\frac{b}{a}\right)$$

$$18) \lim_{x \rightarrow 0} \frac{8^x - e^x}{x} \text{ Usando el ejercicio anterior } \lim_{x \rightarrow 0} \frac{8^x - e^x}{x} = \ln 8 - \ln e = \ln 8 - 1$$

4. Dada la función  $f(x)$  calcular  $f(x+h)$  y calcular el límite  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$1) f(x) = \sqrt{x} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$$2) f(x) = \sqrt[3]{x} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h} \cdot \frac{\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2}} =$$

$$\lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2})} =$$

$$\lim_{h \rightarrow 0} \frac{1}{(\sqrt[3]{(x+h)^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{x^2})} = \frac{1}{(\sqrt[3]{(x+0)^2} + \sqrt[3]{x(x+0)} + \sqrt[3]{x^2})} =$$

$$\frac{1}{\sqrt[3]{x^2}}$$

$$3) f(x) = \frac{1}{x} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{-h}{h} = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = \frac{-1}{x^2}$$

$$4) f(x) = x^2 + 2x + 6 \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) + 6 - (x^2 + 2x + 6)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 6 - x^2 - 2x - 6}{h} = \lim_{h \rightarrow 0} \frac{h^2 + 2xh + 2h}{h} =$$

$$\lim_{h \rightarrow 0} \frac{h(h + 2x + 2)}{h} = \lim_{h \rightarrow 0} h + 2x + 2 = 2x + 2$$

$$5) f(x) = \frac{ax - b}{bx - a} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{a(x+h)-b}{b(x+h)-a} - \frac{ax-b}{bx-a}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{(bx-a)(a(x+h)-b)-(bx+h-a)(ax-b)}{(b(x+h)-a)(bx-a)}}{h} = \lim_{h \rightarrow 0} \frac{(bx-a)(ax+ah-b) - (bx+bh-a)(ax-b)}{h(bx+bh-a)(bx-a)} =$$

$$\lim_{h \rightarrow 0} \frac{(abx^2 + abhx - b^2x - a^2x - a^2h + ab) - (abx^2 + abhx - ax^2 - b^2x - b^2h + ab)}{h(bx+bh-a)(bx-a)} =$$

$$\lim_{h \rightarrow 0} \frac{\frac{(-a^2h) - (-b^2h)}{h(bx+bh-a)(bx-a)}}{h} = \lim_{h \rightarrow 0} \frac{-a^2 - (-b^2)}{(bx+bh-a)(bx-a)} = \frac{-a^2 + b^2}{(bx-a)(bx-a)} =$$

$$-\frac{a^2 - b^2}{(bx-a)^2}$$

$$6) f(x) = \frac{1}{\sqrt{x}} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x(x+h)}}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x(x+h)}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \lim_{h \rightarrow 0} \frac{x - (x+h)}{h\sqrt{x(x+h)}(\sqrt{x} + \sqrt{x+h})} =$$

$$\lim_{h \rightarrow 0} \frac{-h}{h\sqrt{x(x+h)}(\sqrt{x} + \sqrt{x+h})} = \lim_{h \rightarrow 0} \frac{-1}{\sqrt{x(x+h)}(\sqrt{x} + \sqrt{x+h})} =$$

$$\frac{-1}{\sqrt{x(x+0)}(\sqrt{x} + \sqrt{x+0})} = \frac{-1}{\sqrt{x^2}(\sqrt{x} + \sqrt{x})} = -\frac{1}{2x\sqrt{x}}$$

$$7) f(x) = \sqrt{x+3} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}} = \lim_{h \rightarrow 0} \frac{x+h+3 - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})} =$$

$$\lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$$

$$8) f(x) = \sqrt{2x^2 - x} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)^2 - (x+h)} - \sqrt{2x^2 - x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)^2 - (x+h)} - \sqrt{2x^2 - x}}{h} \cdot \frac{\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x}}{\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x}} =$$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h(\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x})} = \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) - x - h - 2x^2 + x}{h(\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x})} =$$

$$\lim_{h \rightarrow 0} \frac{4xh + 2h^2 - h}{h(\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x})} = \lim_{h \rightarrow 0} \frac{h(4x + 2h - 1)}{h(\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x})} =$$

$$\lim_{h \rightarrow 0} \frac{4x + 2h - 1}{\sqrt{2(x+h)^2 - (x+h)} + \sqrt{2x^2 - x}} = \frac{4x + 2(0) - 1}{\sqrt{2(x+(0))^2 - (x+(0))} + \sqrt{2x^2 - x}} =$$

$$\frac{4x - 1}{\sqrt{2x^2 - x} + \sqrt{2x^2 - x}} = \frac{4x - 1}{2\sqrt{2x^2 - x}}$$

$$9) f(x) = \frac{1}{\sqrt{2x-1}} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2(x+h)-1}} - \frac{1}{\sqrt{2x-1}}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{\sqrt{2x-1} - \sqrt{2(x+h)-1}}{\sqrt{2(x+h)-1} \sqrt{2x-1}}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{2x-1} - \sqrt{2(x+h)-1}}{h \sqrt{2(x+h)-1} \sqrt{2x-1}} \cdot \frac{\sqrt{2x-1} + \sqrt{2(x+h)-1}}{\sqrt{2x-1} + \sqrt{2(x+h)-1}} =$$

$$\lim_{h \rightarrow 0} \frac{2x-1 - (2(x+h)-1)}{h \sqrt{2(x+h)-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2(x+h)-1})} =$$

$$\lim_{h \rightarrow 0} \frac{2x-1 - 2x - 2h + 1}{h \sqrt{2(x+h)-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2(x+h)-1})} =$$

$$\lim_{h \rightarrow 0} \frac{-2h}{h \sqrt{2(x+h)-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2(x+h)-1})} =$$

$$\lim_{h \rightarrow 0} \frac{-2}{\sqrt{2(x+h)-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2(x+h)-1})} =$$

$$\frac{-2}{\sqrt{2(x+0)-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2(x+0)-1})} =$$

$$\frac{-2}{\sqrt{2x-1} \sqrt{2x-1} (\sqrt{2x-1} + \sqrt{2x-1})} = \frac{-2}{(2x-1)2\sqrt{2x-1}} = \frac{-1}{(2x-1)\sqrt{2x-1}}$$

$$10) f(x) = \operatorname{sen} x \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen} x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\operatorname{sen} x \cos h + \operatorname{sen} h \cos x - \operatorname{sen} x}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{sen} x(\cos h - 1) + \operatorname{sen} h \cos x}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\operatorname{sen} x(\cos h - 1)}{h} + \frac{\operatorname{sen} h \cos x}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{sen} x(\cos h - 1)}{h} + \lim_{h \rightarrow 0} \frac{\operatorname{sen} h \cos x}{h} =$$

$$\operatorname{sen} x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} + \cos x \lim_{h \rightarrow 0} \frac{\operatorname{sen} h}{h} = \operatorname{sen} x(0) + \cos x(1) = \cos x$$

$$11) f(x) = \log x \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\log(x+h) - \log x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\log\left(\frac{x+h}{x}\right)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \log\left(\frac{x+h}{x}\right) = \lim_{h \rightarrow 0} \log\left(1 + \frac{h}{x}\right)^{\frac{1}{h}} = \lim_{h \rightarrow 0} \log\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{1}{h}} =$$

$$\lim_{h \rightarrow 0} \log\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{1}{h} \cdot \frac{x}{h}} = \lim_{h \rightarrow 0} \log\left(\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}}\right)^{\frac{1}{x}} = \log \lim_{h \rightarrow 0} \left(\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}}\right)^{\frac{1}{x}} =$$

$$\log e^{\frac{1}{x}} = \frac{1}{x} \log e$$

$$12) f(x) = a^x \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{a^{x+h} - a^x}{h}$$

$$\lim_{h \rightarrow 0} \frac{a^x a^h - a^x}{h} = \lim_{h \rightarrow 0} \frac{a^x(a^h - 1)}{h} = a^x \lim_{h \rightarrow 0} \frac{a^h - 1}{h} = a^x \ln a$$

$$13) f(x) = \operatorname{tg} x \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{tg}(x+h) - \operatorname{tg} x}{h}$$

$$\text{Como } \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha \operatorname{tg}\beta} \Rightarrow \operatorname{tg}\alpha - \operatorname{tg}\beta = (1 + \operatorname{tg}\alpha \operatorname{tg}\beta)(\operatorname{tg}(\alpha - \beta))$$

$$\lim_{h \rightarrow 0} \frac{\operatorname{tg}(x+h) - \operatorname{tg} x}{h} = \lim_{h \rightarrow 0} \frac{(1 + \operatorname{tg}(x+h) \operatorname{tg} x) \operatorname{tg}(x+h-x)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{(1 + \operatorname{tg}(x+h) \operatorname{tg} x) \operatorname{tg} h}{h} = \lim_{h \rightarrow 0} (1 + \operatorname{tg}(x+h) \operatorname{tg} x) \frac{\operatorname{tg} h}{h} = (1 + \operatorname{tg}^2 x)(1) =$$

$$14) f(x) = \ln x \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln x}{h}$$

$$\text{Por ejercicio resuelto con anterioridad } \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln x}{h} = \frac{1}{x} \ln e = \frac{1}{x}(1) =$$

$$\frac{1}{x}$$

$$15) f(x) = \ln(2x-1) \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\ln(2(x+h)-1) - \ln(2x-1)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\ln\left(\frac{2x+2h-1}{2x-1}\right)}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \ln\left(\frac{2x-1+2h}{2x-1}\right) = \lim_{h \rightarrow 0} \frac{1}{h} \ln\left(1 + \frac{2h}{2x-1}\right) =$$

$$\lim_{h \rightarrow 0} \ln\left(1 + \frac{2h}{2x-1}\right)^{\frac{1}{h}} = \lim_{h \rightarrow 0} \ln\left(1 + \frac{1}{\frac{2x-1}{2h}}\right)^{\frac{2x-1}{2h}} \frac{2}{2x-1} = \ln e^{\lim_{h \rightarrow 0} \frac{2}{2x-1}} =$$

$$\ln e^{\frac{2}{2x-1}} = \frac{2}{2x-1} \ln e = \frac{2}{2x-1}$$

16)  $f(x) = \sin(3x+c)$   $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sin(3(x+h)+c) - \sin(3x+c)}{h}$

$$\lim_{h \rightarrow 0} \frac{\sin(3(x+h)+c) - \sin(3x+c)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{3(x+h)+c-(3x+c)}{2}\right) \cos\left(\frac{3(x+h)+c+(3x+c)}{2}\right)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{3x+3h+c-3x-c}{2}\right) \cos\left(\frac{3x+3h+c+3x+c}{2}\right)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{3h}{2}\right) \cos\left(\frac{6x+3h+2c}{2}\right)}{h} = \lim_{h \rightarrow 0} \frac{\sin\left(\frac{3h}{2}\right) \cos\left(\frac{6x+3h+2c}{2}\right)}{\frac{h}{2}} =$$

$$\lim_{h \rightarrow 0} \frac{3 \sin\left(\frac{3h}{2}\right) \cos\left(\frac{6x+3h+2c}{2}\right)}{\frac{3h}{2}} = \lim_{h \rightarrow 0} 3 \frac{\sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \cos\left(\frac{6x+3h+2c}{2}\right) =$$

$$(3)(1) \cos\left(\frac{6x+3(0)+2c}{2}\right) = (3) \cos\left(\frac{6x+2c}{2}\right) = 3 \cos(3x+c)$$

17)  $f(x) = \operatorname{arctg} x$   $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{arctg}(x+h) - \operatorname{arctg} x}{h}$

Sea  $\alpha = \operatorname{arctg}(x+h)$  y  $\beta = \operatorname{arctg} x \Rightarrow \operatorname{tg} \alpha = x+h$  y  $\operatorname{tg} \beta = x$

Sea  $\alpha - \beta = t \Rightarrow \operatorname{tg} t = \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} = \frac{x+h-x}{1 + (x+h)x} = \frac{h}{1+x^2+hx} \Rightarrow$

$$\operatorname{tg} t = \frac{h}{1+x^2+hx} \Rightarrow t = \operatorname{arctg}\left(\frac{h}{1+x^2+hx}\right)$$

$$\lim_{h \rightarrow 0} \frac{\operatorname{arctg}(x+h) - \operatorname{arctg} x}{h} = \lim_{h \rightarrow 0} \frac{\operatorname{arctg}\left(\frac{h}{1+x^2+hx}\right)}{h} =$$

$$\lim_{h \rightarrow 0} \frac{\operatorname{arctg}\left(\frac{h}{1+x^2+hx}\right)}{\frac{h}{1+x^2+hx}} \frac{1}{1+x^2+hx} = (1) \lim_{h \rightarrow 0} \frac{1}{1+x^2+hx} = \frac{1}{1+x^2+0x} =$$

$$\frac{1}{1+x^2}$$

$$\begin{aligned}
18) \quad f(x) &= \frac{1}{\sqrt[3]{x}} \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt[3]{x+h}} - \frac{1}{\sqrt[3]{x}}}{h} \\
&\lim_{h \rightarrow 0} \frac{\sqrt[3]{x} - \sqrt[3]{x+h}}{h \sqrt[3]{x+h} \sqrt[3]{x}} = \lim_{h \rightarrow 0} \frac{\sqrt[3]{x} - \sqrt[3]{x+h}}{h \sqrt[3]{x^2 + hx}} = \\
&\lim_{h \rightarrow 0} \frac{\sqrt[3]{x} - \sqrt[3]{x+h}}{h \sqrt[3]{x^2 + hx}} \quad \frac{\sqrt[3]{x^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{(x+h)^2}}{\sqrt[3]{x^2} + \sqrt[3]{x(x+h)} + \sqrt[3]{(x+h)^2}} = \\
&\lim_{h \rightarrow 0} \frac{x - x - h}{(h \sqrt[3]{x^2 + hx})(\sqrt[3]{x^2} + \sqrt[3]{x(x+h)}) + \sqrt[3]{(x+h)^2}} = \\
&\lim_{h \rightarrow 0} \frac{-h}{(h \sqrt[3]{x^2 + hx})(\sqrt[3]{x^2} + \sqrt[3]{x(x+h)}) + \sqrt[3]{(x+h)^2}} = \\
&\lim_{h \rightarrow 0} \frac{-1}{(\sqrt[3]{x^2 + hx})(\sqrt[3]{x^2} + \sqrt[3]{x(x+h)}) + \sqrt[3]{(x+h)^2}} = \frac{-1}{(\sqrt[3]{x^2})(3\sqrt[3]{x^2})} = \frac{-1}{3\sqrt[3]{x^4}}
\end{aligned}$$