



## Cálculo integral

**Integración por fracciones parciales  
Integral definida**

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# Factores Lineales

Existen tres casos, sólo nos fijamos en el denominador para identificar el caso.

- Factores lineales distintos

$$\int \frac{ex^2 + fx + g}{(x - a_0)(x - a_1) \cdots (x - a_n)} dx$$

- Factores lineales repetidos

$$\int \frac{ex^2 + fx + g}{(x - a_0)(x - a_1)^2 \cdots (x - a_n)^n} dx$$

- Factores cuadráticos diferentes

$$\int \frac{ex^2 + fx + g}{(x^2 - a_0)(x^2 - a_1) \cdots (x^2 - a_n)} dx$$

➤ Factores lineales distintos

$$\frac{dx^2 + ex + f}{(x - a_0)(x - a_1) \cdots (x - a_n)} = \frac{A_0}{(x - a_0)} + \frac{A_1}{(x - a_1)} + \cdots + \frac{A_n}{(x - a_n)}$$

➤ Factores lineales repetidos

$$\frac{dx^2 + ex + f}{(x - a_0)(x - a_1)^2 \cdots (x - a_n)^n} = \frac{A_0}{(x - a_0)} + \frac{A_1}{(x - a_1)^2} + \cdots + \frac{A_n}{(x - a_n)^n}$$

➤ Factores cuadráticos diferentes

$$\frac{dx^2 + ex + f}{(x^2 - a_0)(x^2 - a_1) \cdots} = \frac{A_0x + B_0}{(x^2 - a_0)} + \frac{A_1x + B_1}{(x^2 - a_1)} + \cdots + \frac{A_nx + B_n}{(x^2 - a_n)}$$

# Factores lineales distintos

$$\int \frac{x-3}{(x^2-x)} dx = \int \frac{x-3}{x(x-1)} dx = \int \frac{A}{x} dx + \int \frac{B}{x-1} dx$$

Resolución

$$\frac{x-3}{x(x-1)} = \frac{A}{x} + \frac{B}{(x-1)}$$

$$x-3 = \frac{Ax(x-1)}{x} + \frac{Bx(x-1)}{(x-1)}$$

$$x-3 = A(x-1) + Bx$$

Se asigna un valor a X=1

$$1-3 = A(1-1) + B(1)$$

$$-2 = 0 + B \quad B = -2$$

Se asigna un valor a X=0

$$0-3 = A(0-1) + B(0)$$

$$A = 3$$

Sustituyendo los valores de A y B en la integral

$$\begin{aligned} &= \int \frac{A}{x} dx + \int \frac{B}{x-1} dx = \int \frac{3}{x} dx + \int \frac{-2}{x-1} dx \\ &= \ln|x| - 2\ln|x-1| + c \end{aligned}$$

$$= \ln|x| + \ln|x-1|^{-2} + c$$

# Factores lineales repetidos

$$\int \frac{x+5}{(x+1)^2} dx = \int \frac{x+5}{(x+1)(x+1)^2} dx = \int \frac{A}{(x+1)} dx + \int \frac{B}{(x+1)^2} dx$$

Resolución

$$\frac{x+5}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$$

$$x+5 = \frac{A(x+1)^2}{(x+1)} + \frac{B(x+1)^2}{(x+1)^2}$$

$$x+5 = A(x+1) + B$$

Se asigna un valor a  $x=-1$

$$-1+5 = A(-1+1) + B$$

$$4 = B$$

Se igualan términos semejantes

$$x+5 = A(x+1) + B$$

$$x+5 = Ax + A + B$$

$$x = Ax \quad \rightarrow \quad 5 = A + B$$

Se sustituye el valor de B y se obtiene el Valor de A

$$5 = A + 4 \quad \rightarrow \quad 5 - 4 = A \quad \rightarrow \quad A = 1$$

Sustituyendo los valores de A y B en la integral

$$= \int \frac{A}{(x+1)} dx + \int \frac{B}{(x+1)^2} dx$$

$$= \int \frac{1}{(x+1)} dx + \int \frac{4}{(x+1)^2} dx$$

$$= \ln|x+1| - \frac{4}{(x+1)} + C$$

Ejemplo: determinar la descomposición en fracciones parciales de la integral  $\int \frac{2}{x^2-2x} dx$

Resolución: Caso 1. factores lineales diferentes

a)  $\int \frac{-1}{x} dx + \int \frac{-1}{x-2} dx$

b)  $\int \frac{1}{x} dx + \int \frac{1}{x-2} dx$

c)  $\int \frac{-1}{x} dx + \int \frac{1}{x-2} dx$

d)  $\int \frac{1}{x} dx + \int \frac{-1}{x-2} dx$

$$\int \frac{2}{x^2-2x} dx = \int \left( \frac{A}{x} + \frac{B}{x-2} \right) dx$$

$$\frac{2}{x^2-2x} = \frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$\left( \frac{2}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2} \right) x(x-2)$$

$$\frac{2x(x-2)}{x^2-2x} = \frac{Ax(x-2)}{x} + \frac{Bx(x-2)}{x-2}$$

$$2 = A(x-2) + Bx$$

$$2 = Ax - 2A + Bx$$

$$2 = (A+B)x - 2A$$

$$\begin{aligned} A + B &= 0 \\ -2A &= 2 \end{aligned} \rightarrow A = \frac{-1 + B}{2} = -1 \rightarrow \begin{aligned} B &= 1 \\ A &= -1 \end{aligned}$$

$$= \int \left( \frac{A}{x} + \frac{B}{x-2} \right) dx = \int \frac{-1}{x} dx + \int \frac{1}{x-2} dx = -1 \int \frac{dx}{x} + \int \frac{dx}{x-2} = -\ln|x| + \ln|x-2| + c$$

# Integral definida

## Teorema fundamental del cálculo

- $\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$
- Teoremas de la integral definida
  - $\int_a^b f(x)dx = - \int_b^a f(x)dx$
  - $\int_{-a}^a f(x)dx = 2 \int_{-a}^0 f(x)dx = 2 \int_0^a f(x)dx$
  - $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$



Determinar la integral siguiente:  $\int_{\frac{1}{2}}^{\frac{3}{2}} \left( \frac{x}{2} - 1 \right) dx$

### Resolución

a)  $-\frac{64}{11}$

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \left( \frac{x}{2} - 1 \right) dx = \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{x}{2} dx - \int_{\frac{1}{2}}^{\frac{3}{2}} dx = \frac{x^2}{4} - x$$

b)  $-\frac{11}{64}$

$$= \left( \frac{\left(\frac{3}{4}\right)^2}{4} - \frac{3}{4} \right) - \left( \frac{\left(\frac{1}{2}\right)^2}{4} - \frac{1}{2} \right) = \left( \frac{9}{16} - \frac{3}{4} \right) - \left( \frac{1}{4} - \frac{1}{2} \right) =$$

c)  $-\frac{11}{64}$

$$= \left( \frac{9}{64} - \frac{3}{4} \right) - \left( \frac{1}{16} - \frac{1}{2} \right) = \left( \frac{9}{64} - \frac{48}{64} \right) - \left( \frac{4}{64} - \frac{32}{64} \right)$$

d)  $\frac{64}{11}$

$$= \left( \frac{-39}{64} \right) - \left( \frac{-28}{64} \right) = \frac{-39 + 28}{64} = \frac{-11}{64}$$

Calcular la integral siguiente:  $\int_{-1}^3 (3x^2 + 5x - 1)dx$

a) 44

$$\int_{-1}^3 (3x^2 + 5x - 1)dx = \left( \frac{3x^3}{3} + \frac{5x^2}{2} - x \right) \Big|_{-1}$$

b) 42

$$= \left( \frac{3(3)^3}{3} + \frac{5(3)^2}{2} - (3) \right) - \left( \frac{3(-1)^3}{3} + \frac{5(-1)^2}{2} - (-1) \right)$$

c) 41

$$= \left( \frac{81}{3} + \frac{45}{2} - (3) \right) - \left( \frac{-3}{3} + \frac{5}{2} - (-1) \right)$$

d) 40

$$= \left( \frac{54+45-6}{2} \right) - \left( \frac{-2+5+2}{2} \right) = \frac{88}{2} = 44u^2$$